Discounts for Lack of Marketability: What the Scientific Method Tells Us

By Marc Vianello, CPA, ABV, CFF

Observe. Hypothesize. Test. These are the principles of the scientific method. The scientific method seeks answers to questions raised from observation, formulates hypotheses based on the observations and questions, makes predictions regarding the hypotheses, and tests the hypotheses. A scientific hypothesis must be falsifiable to have a valid test, meaning that it must be possible to disprove the hypothesis. A hypothesis that is impossible to disprove is inherently unreliable, because it cannot be confirmed. Conversely, a hypothesis that is falsified is proved to be wrong.

This article uses the scientific method to assess the reliability of the various methods for estimating discounts for lack of marketability (DLOM). The article considers the principal methods of estimating DLOM, including benchmarking with restricted stock and IPO transactions; the Pluris and FMV/Stout DLOM calculators; the Finnerty, Black-Scholes, and Longstaff option models; and probability-based Finnerty, Black-Scholes, and Longstaff option models. It seeks to identify DLOM methodologies that can survive testing using the scientific method.

Valuation practice regarding DLOMs has historically relied on implied hypotheses that in and of themselves cannot be falsified. For example, practitioners may intuitively hypothesize that the following price difference observations represent DLOM because they may reflect a degree of lack of marketability:[1]

1. The differences between restricted stock prices and their contemporaneous unrestricted stock prices.
2. The differences between pre-IPO stock prices and their noncontemporaneous IPO stock prices.
3. The “cost of flotation,” meaning the cost of achieving marketability by means of a public offering.
4. Long-term equity anticipation securities (LEAPS), which represent the percentage cost of acquiring protective puts for publicly traded stocks. The percentage cost is the put cost of a publicly traded stock divided by the stock price.

1 *The International Glossary of Business Valuation Terms* defines “marketability” as “the ability to quickly convert property to cash at minimal cost,” and “discount for lack of marketability” as “an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability.” The *International Glossary of Business Valuation Terms* was adopted in 2001 by the American Institute of Certified Public Accountants, the American Society of Appraisers, the Canadian Institute of Chartered Business Valuators, the National Association of Certified Valuation Analysts, and The Institute of Business Appraisers. *Discount for Lack of Marketability—Job Aid for IRS Valuation Professionals*, September 25, 2009, page 5, explains the difference between “liquidity” and “marketability”: “If it’s liquid, it’s marketable; If it’s non-marketable, it’s illiquid; Being illiquid does not [necessarily] mean non-marketable—it may still be sellable but not quickly or without loss of value.”

A hypothesis that any one of the above price observation sets does or does not predict DLOM is not falsifiable in isolation, because it cannot be shown that any of the observations represent DLOM and only DLOM.[2] We simply do not know what other ingredients influenced the price differentials, and we do not know, in isolation, what DLOM should be.

If the above-listed observations represent DLOM, they should yield statistically correlated results among themselves via linear regression, assuming that the transactions can be matched. Anecdotally they seem not to—practitioners usually produce very different results from the various transactional sources. If the linear regressions of any two of the different discount observations sufficiently correlate, then one could perhaps conclude that they are somehow related and predict DLOM. But the distinct types of transactions—with different measurement of the underlying transactions coupled with known and unknown ingredients, such as premiums, compensation, relationship biases, changes in holding periods, the size of the stock block, and changes in economic conditions, among others—suggest that the observed results will not correlate to a reasonable degree of statistical reliability.

It would be pointless to compare the discounts of the rare transactions that overlap the Pluris and FMV/Stout databases; any discount difference would be due to discount measurement and would tell us nothing about DLOM measurement. Meanwhile, regression analysis cannot be done for restricted stock transactions versus pre-IPO transactions versus flotation costs versus LEAPS transactions unless the opposing transactions can be matched—a seemingly impossible task. The different price observations of restricted stock discounts, pre-IPO transactions, cost of flotation, and LEAPS therefore contradict their general use for benchmarking a reliable DLOM for a specific valuation subject.[3]

The Benchmarking Methodologies

It is problematic for the valuation community that a positive hypothesis favoring DLOM benchmarking based on the above price observations can easily be falsified. The price differences of the benchmarked transactions can usually be shown to include known ingredients that inherently are not DLOM. Any such showing falsifies the hypothesis. Meanwhile, the null hypothesis (i.e., that the benchmarking does not represent DLOM) cannot be falsified without knowing theoretically correct DLOM numbers. The result is that benchmarking based directly on restricted stocks, pre-IPOs, flotation costs, or LEAPS should be considered unreliable conjecture.[4]

Additionally confounding the use of the available transactional databases for DLOM benchmarking is the limited number of transactions that might closely approximate the valuation subject. You can read about these limitations in my book, *Empirical Research* Regarding *Discounts for Lack of Marketability*, available free at https://dlomcalculator.com. Chapters 4 and 5 pertain to the restricted stock transactions reported by Pluris and FMV/Stout, respectively, and show the general lack of statistical significance of the relationship of the database companies' metrics and the observed transactional discounts.[5]

The Mandelbaum Criteria

The U.S. Tax Court uses the *Mandelbaum*[6] criteria to assess the reasonableness of DLOM estimates. The affirmation or rejection of a separately developed DLOM conclusion is an entirely acceptable use of the criteria. But the criteria do not directly yield a DLOM percentage. Using the criteria to directly estimate a DLOM is guessing. For that reason, a hypothesis that the *Mandelbaum* criteria directly yield a reliable DLOM is not falsifiable, and is an inherently unreliable use.

The Calculator Methodologies

Many practitioners intuitively hypothesize that other methodologies result in reasonable DLOM estimates, including:

1. The Quantitative Marketability Discount Model (QMDM), which is a spreadsheet methodology.
2. Calculators based on pre-IPO transactions.
3. The Pluris and FMV/Stout calculators, which are based on their databases of restricted stock transactions.
4. Options-based formulas, such as Finnerty, Black-Scholes, and Longstaff, among others.
5. Probability-based DLOM, using the Finnerty, Black-Scholes, or Longstaff formula in conjunction with the VFC DLOM Calculator.[7]

2 Other price difference observations, such as those by Bajaj and Abbott, have been suggested to represent a lack of liquidity, but not necessarily to represent DLOM.

3 An interesting study would be to attempt comparison of contemporaneous LEAPS percentage costs to restricted stock percentage discounts of the same stock. The result might falsify hypotheses that one does or does not predict the other. I have not made this study. I invite other researchers to undertake the task and to publish their results.

4 This deficiency is not ameliorated by using an "all of the above," "salad bowl" approach to DLOM estimation: garbage in, garbage out.

5 *Empirical Research* reports that the reported restricted stock discounts most closely correlate with the price volatility of the issuers' publicly traded stocks. See pages 43, 46, 49, and 50 re Pluris; pages 88–92 re FMV/Stout.

6 Mandelbaum v. Commissioner, T.C. Memo 1995-255 (U.S. Tax Ct. 1995).

7 The VFC DLOM Calculator is located at https://dlomcalculator.com.

Let us apply the discussion of the preceding paragraphs to these methodologies:

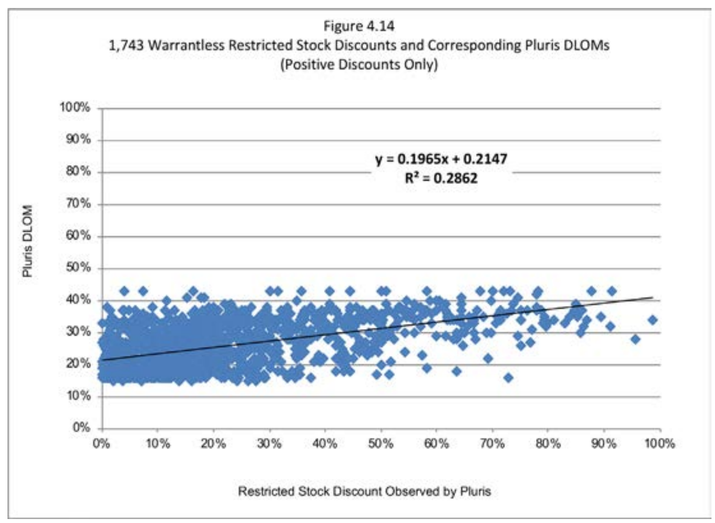
- The hypothesis to scientifically test QMDM would be, “The QMDM methodology predicts (or does not predict) the [what?] underlying discounts.” Neither hypothesis can be scientifically tested because QMDM is not based on a set of transactions or data from which independent DLOM conclusions are derived. QMDM DLOM calculations are the “educated” guess of a single person deemed to have better judgment than all others. This makes QMDM an unreliable means of calculating DLOM.
- The pre-IPO, Pluris, and FMV/Stout calculators have underlying transaction databases against which their calculators can be statistically tested. Ignoring that the price differences may be (are?) due to more than DLOM, these calculators should predict the underlying observations. This would be the start of a scientific test.
- Proving that the Finnerty, Black-Scholes, and Longstaff formulas yield reliable DLOMs is not possible in isolation. The formulas are theoretical models that do not draw from actual transactions. More is needed to test them scientifically. It should be understood, however, that only the Longstaff formula was created to predict DLOM. The others were created to price options for investment purposes.

A hypothesis to test the database and formula DLOM calculation methodologies (excluding QMDM) is, “The [specified] DLOM methodology predicts the corresponding [specified] observed discounts.” This hypothesis is conceptually falsifiable, but defective. First, the hypothesis impliedly assumes that the observed discounts represent DLOM. As discussed, that assumption is easily falsified. Second, a linear regression with an R-square of 1 percent offers a prediction that may be statistically measurable but leaves 99 percent of the correlation unexplained. It therefore makes sense to specify a minimum regression standard for falsifying the hypothesis.

Setting aside the implication that the observed discounts necessarily represent DLOM, a more tightly crafted hypothesis is, “The [specified] methodology predicts the corresponding [specified] observed discounts with a linear regression relationship exhibiting these characteristics: (1) x-y intercept ~ zero; (2) coefficient of x ~ y; and (3) R-square > 50 percent.” This parameter-based hypothesis is clearly falsifiable, but is still not sufficient because a single negative result does not preclude the possibility of false positives that satisfy the parameters. The better hypothesis is stated in the negative (the “null hypothesis”): “The [specified] methodology does not predict the corresponding [specified] observed discounts with a linear regression relationship exhibiting these characteristics: (1) x-y intercept ~ zero; (2) coefficient of x ~ y; and (3) R-square > 50 percent.” A single result satisfying the parameters conclusively falsifies the null hypothesis even if contradictory results are found. Multiple tests falsifying the null hypothesis add substantiation to the conclusion that the method predicts the result.

A. The Database Calculators

Chapters 4 and 5 of *Empirical Research* discuss correlation tests performed on the Pluris and FMV/Stout calculator results with their respective databases. Excluding the discount-distorting restricted stock transactions with price premiums or warrants (premiums clearly are not DLOM; the warrants distort discount measurement), *Empirical Research* Figure 4.14 shows the weak regression results of the Pluris DLOM calculator versus the corresponding observed discounts of its database. I could not find a legitimate set of Pluris calculator DLOMs and corresponding restricted stock discounts that falsified the null hypothesis.[8]



Source: VFC DLOM Calculator, www.dlomcalculator.com

Table 1 reports the weak correlation of the FMV/Stout calculator versus the observed discounts of the corresponding 638 transactions. Again, I was unable to negate the null hypothesis. Only a group of 18 and a group of five transactions were found to have statistical correlation greater than 50 percent, and only the group of five had desirable formula attributes with a line slope approximating 1.0 and the x-y intercept approximating 0 percent. But the perfect correlation of the five-transaction group is excludable because it merely proves the regression methodology—any group of five divided into quintiles will result in a perfect correlation.

8 It would be possible to select a bottom-up set of transactions with discounts within the range of the Pluris (or FMV/Stout) calculator limitations to achieve satisfactory correlation results. This would be illegitimate, however, because the results would have been manufactured. A legitimate test requires top-down selection.

Table 1: Correlation of Median Quintile DLOMs with Positive Discount Transactions

Rule 144 Time Period	Registration Rights	Transaction Count	Average Transaction Discount	Average Stout DLOM	Regression Line Slope	Y-Axis Intercept	R-Square
All	All	638	21%	18%	1.926	-13.5%	26%
2 Years	No	178	24%	22%	1.714	-14.1%	29%
1 Year	Yes	159	17%	14%	2.420	-16.5%	30%
1 Year	Blank	121	27%	22%	2.067	-18.3%	25%
6 Months	Yes	94	15%	12%	4.230	-34.3%	15%
2 Years	Yes	40	24%	24%	3.391	-56.0%	45%
1 Year	No	23	25%	23%	1.436	-7.9%	22%
6 Months	No	18	17%	17%	2.155	-19.5%	60%
2 Years	Blank	5	23%	23%	1.000	0.0%	100%

These test results undermine using the Pluris and FMV/Stout calculators to reach reliable DLOM conclusions. The tests show that the calculators do not falsify the null hypothesis, and do not replicate the underlying observations. It may be that tests performed by other persons will produce different results, but until then one must conclude that there is no scientific basis for stating that the calculators reliably predict the underlying transactions even if one were to accept, erroneously, that the populations of observed discounts represent DLOM.

I did not perform similar analyses of pre-IPO calculators and their underlying transactions using the null hypothesis. Others are invited to do so and to publish the results.

B. The Formulas and Formula Calculators

I used three forms of simulation-derived data to test the Finnerty, Black-Scholes, and Longstaff formulas. First, I used a cleansed population of restricted stock discounts derived from the Pluris and FMV/Stout restricted stock databases. The cleansing process is discussed below. Second, I obtained the daily stock closing prices for the restricted stock issuers for time periods ending with the date the restricted stocks were issued. I calculated and considered price volatility means and standard deviations for a variety of pre-issue time periods, as discussed below. Third, I measured the time it took the SEC to approve S-1 filings on a standard industrial classification (SIC) basis. The derived information allowed me to scientifically address the Finnerty, Black-Scholes, and Longstaff formulas using the null hypothesis, "The [specified] formula *does not predict* the corresponding observed restricted stock discounts with a linear regression relationship exhibiting these characteristics: (1) x-y intercept ~ zero; (2) coefficient of x ~ y; and (3) R-square > 50 percent." I refer to this as the "Formulas Null Hypothesis." The Finnerty and Black-Scholes formulas include a risk-free rate parameter, and the Black-Scholes formula includes a dividend yield parameter. The tests discussed in this article assume zero for these parameters. See *Empirical Research*, Chapter 7, for more discussion of the zero assumptions.

- I tested the formulas with the static price volatilities reported by Pluris and FMV/Stout and 90, 180, and 360 days as static time variables. None of the R-squares of correlation exceeded 6.5 percent using the Pluris and FMV/Stout population of 4,372 transactions for which the databases reported price volatility. The Formulas Null Hypothesis was therefore not falsified.
- Zero or negative discounts are reported for many of the transactions in the Pluris and FMV/Stout databases. Removing those reduced the population to 3,869 transactions. I performed the same 90-, 180-, and 360-day marketing period tests of this population. The range of R-squares of correlation for these tests ranged from 18.32–25.38 percent. The Formulas Null Hypothesis was again not falsified.

The preceding paragraph shows that removing statistical noise has a positive effect on the regression results. Further, the other non-DLOM characteristics affect the restricted stock discounts reported by Pluris and FMV/Stout. For example, the warrant-tainted stocks in the Pluris database distort the reported transaction discounts. These and other transaction characteristics can cause the x-y intercept to deviate from zero—a widespread problem in empirical research. Culling the population of restricted stock transactions in the databases to remove such statistical noise is necessary.

Table 2 shows the initial culling process based on (a) the Pluris and FMV/Stout reported price volatilities, and (b) the recalculated mean price volatilities based on available reported daily closing prices.

Table 2: Population Refinement Improves the Relationship of Restricted Stock Discounts to Price Volatilities[9]

	FMV/Stout and Pluris® Reported Volatilities		VFC DLOM Calculator® Average Price Volatilities	
	Number of Transactions	Logarithmic Regression	Number of Transactions	Logarithmic Regression
All Stout Study (769) and Pluris® (3,632) restricted stock transactions	4,401		4,401	
Transactions with no price volatility reported by Pluris® and Stout	(29)		n/a	
	4,372	$R^2 = 0.0622$	4,401	
Pluris® transactions with warrants reported	(1,867)		(1,867)	
	2,505	$R^2 = 0.0384$	2,534	
Transactions closing dates prior to September 15, 2007 (price history not available)	n/a		(1,687)	
	2,505	$R^2 = 0.0384$	847	
Issuers apparently no longer publicly traded	n/a		(427)	
	2,505[10]	$R^2 = 0.0384$	420	
Issuers with zero percent price volatility	n/a		(13)	
	2,505	$R^2 = 0.0384$	407[11]	$R^2 = 0.0182$
FMV/Stout duplicates for which Pluris® has warrants	(2)		(1)	
	2,503	$R^2 = 0.0384$	406	$R^2 = 0.018$
Pluris® transactions with FMV/Stout duplicate (priority was given to FMV/Stout transactions)	(196)		(48)	
	2,307[12]	$R^2 = 0.0359$	358[13]	$R^2 = 0.0144$
Transactions with zero or negative discounts	(382)		(91)	
Positive discount transactions with price volatilities (excludes duplicates)	1,925	$R^2 = 0.2348$	267	$R^2 = 0.0898$
Issuer stock prices that failed VFC's price verification test	n/a		(67)	
Refined restricted stock issuer dataset	1,925[14]	$R^2 = 0.2348$	200[15]	$R^2 = 0.179$

9 The FMV/Stout and Pluris columns use the one-year price volatilities for the transactions as reported by those databases. The VFC DLOM Calculator columns use the restricted stock issuer's average price volatilities calculated by the VFC DLOM Calculator for the 250 days preceding the applicable transaction closing date.

10 Using linear regression, this group of transactions has a t Stat of 8.8 and a P-value of 2.8E-18. The relationship is statistically significant.

11 Using linear regression, this group of transactions has a t Stat of 2.5 and a P-value of 0.0127. The relationship is statistically significant.

12 Using linear regression, this group of transactions has a t Stat of 8.2 and a P-value of 4.0E-16. The relationship is statistically significant.

13 Using linear regression, this group of transactions has a t Stat of 2.1 and a P-value of 0.0345. The relationship is statistically significant.

14 Using linear regression, this group of transactions has a t Stat of 17.9 and a P-value of 3.2E-66. The relationship is statistically significant.

15 Using linear regression, this group of transactions has a t Stat of 7.9 and a P-value of 1.7E-13. The relationship is statistically significant.

Table 3 shows the second round of culling based on (a) the registration rights reported by Pluris and FMV/Stout, (b) the stock issuers' SIC codes, and (c) excluding the years of and before the Great Recession. The combined culling resulted in a population of 59 "clean" restricted stock transactions. Table 3 shows the improvement in DLOM regression results as the dataset was refined.

Table 3: Secondary Refinement of the Test Dataset

Number of Restricted Stock Transactions	Closing Date Range	Number of SEC Approvals in the Issuers' 4-Digit SIC Codes	Transaction Discount	Registration Rights	Linear Regressions vs. Transaction Discounts					
					VFC Longstaff DDLOM*			VFC Black-Scholes DDLOM*		
					Slope	Intercept	R-Square	Slope	Intercept	R-Square
Refined restricted stock issuer dataset with VFC calculated price volatility probabilities										
200 per Table 2	2007–2014	n/a	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	DLOMs could not be calculated for 55 transactions because the issuers' reported 4-digit SIC code could not be found in the VFC database of SEC filings.					
R-squares of correlation and regression formulas improve with more specific SIC codes, when transactions with unknown registration rights are removed, and when the Great Recession years are removed										
145	2007–2014	1 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7520	5.20%	19.93%	1.6415	4.86%	23.79%
140	2007–2014	2 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7738	4.29%	21.23%	1.6872	3.95%	25.41%
130	2007–2014	3 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8334	3.35%	24.28%	1.8037	3.10%	28.77%
118	2007–2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8984	2.15%	26.94%	1.9796	1.61%	31.98%
75	2010–2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	1.0612	-0.19%	35.86%	2.2480	-0.29%	41.49%
59	2010–2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	1.0109	-2.84%	54.19%	2.0769	-2.08%	57.45%

* DDLOM refers to double-probability DLOM.

Returning briefly to subsection A, above, the FMV/Stout and Pluris calculator DLOMs were retested using this 59-transaction population. The calculator results had only a 16.1 percent R-square of correlation with the observed discounts of the transactions, again failing to falsify their null hypotheses.

Continuing with the Finnerty, Black-Sholes, and Longstaff formulas, these were tested using the 59-transaction population; the price volatilities reported by Pluris and FMV/Stout; and 90-, 180-, and 360-day static marketing periods (see Table 4).[16] These tests were statistically significant. However, the x-coefficients of the regression lines failed to approximate 1.0, and the highest R-square of correlation was 42.88 percent. The tests failed to falsify the Formulas Null Hypothesis.

Table 4: Regression Results for 59 Pluris and FMV/Stout Restricted Stock Transactions Finnerty, Black-Scholes, and Longstaff Formula DLOMs[17] Using the Database Price Volatilities[18] and 90, 180, and 360 Days to Sale Date

	Formula	R-Square
Finnerty: Database volatility, StdDev = 0; Marketing mean 90 days, StdDev = 0	y = 3.2548x + 0.0018	R^2 = 0.4193
Black-Scholes: Database volatility, StdDev = 0; Marketing mean 90 days, StdDev = 0	y = 0.8346x + 0.0134	R^2 = 0.4288
Longstaff: Database volatility, StdDev = 0; Marketing mean 90 days, StdDev = 0	y = 0.3797x + 0.0092	R^2 = 0.3954
Finnerty: Database volatility, StdDev = 0; Marketing mean 180 days, StdDev = 0	y = 1.9218x - 0.0158	R^2 = 0.3920
Black-Scholes: Database volatility, StdDev = 0; Marketing mean 180 days, StdDev = 0	y = 0.6389x + 0.0054	R^2 = 0.4235
Longstaff: Database volatility, StdDev = 0; Marketing mean 180 days, StdDev = 0	y = 0.3019x + 0.0010	R^2 = 0.2871
Finnerty: Database volatility, StdDev = 0; Marketing mean 360 days, StdDev = 0	y = 1.1856x - 0.0362	R^2 = 0.3314
Black-Scholes: Database volatility, StdDev = 0; Marketing mean 360 days, StdDev = 0	y = 0.5140x - 0.0080	R^2 = 0.4092
Longstaff: Database volatility, StdDev = 0; Marketing mean 360 days, StdDev = 0	y = 0.2362x + 0.0023	R^2 = 0.1691

Table 5: Regression Results for 59 Pluris and FMV/Stout Restricted Stock Transactions Finnerty, Black-Scholes, and Longstaff Formula DLOMs Using VFC-Calculated 250-Day Average Price Volatilities and 90, 180, and 360 Days to Sale Date

	Formula	R-Square
Finnerty: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 90 days, Std Dev = 0	y = 6.7043x - 0.0200	R^2 = 0.5751
Black-Scholes: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 90 days, Std Dev = 0	y = 1.8632x - 0.0159	R^2 = 0.5762
Longstaff: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 90 days, Std Dev = 0	y = 0.7149x + 0.0084	R^2 = 0.5804
Finnerty: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 180 days, Std Dev = 0	y = 3.5591x - 0.0280	R^2 = 0.5707
Black-Scholes: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 180 days, Std Dev = 0	y = 1.3517x - 0.0193	R^2 = 0.5752
Longstaff: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 180 days, Std Dev = 0	y = 0.5836x - 0.0162	R^2 = 0.5917
Finnerty: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 360 days, Std Dev = 0	y = 1.9865x - 0.0427	R^2 = 0.5526
Black-Scholes: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 360 days, Std Dev = 0	**y = 1.0022x - 0.0254**	**R^2 = 0.5721**
Longstaff: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 360 days, Std Dev = 0	y = 0.4409x - 0.0278	R^2 = 0.5157

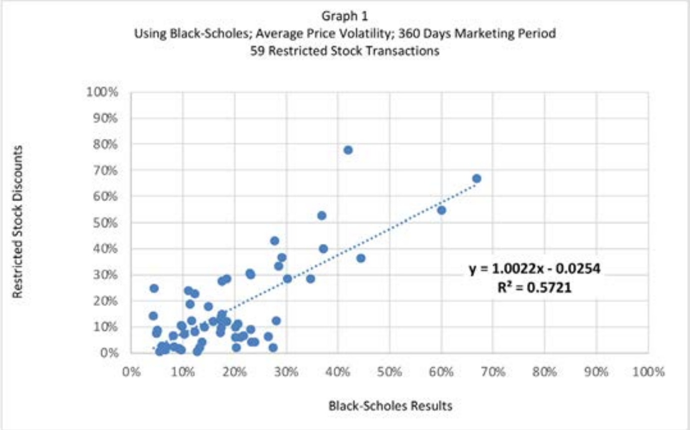
16 *See* Exhibit 1 for the transaction details and regression statistics. Exhibit 1 and the other exhibits referred to below can be found at https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf.

17 As applicable, the risk-free rate and dividend yield variables in the Black-Scholes and Finnerty formulas are assumed to be zero. A criticism of the Longstaff formula as conventionally applied is that it can yield values greater than 100 percent, with high volatility and long time-period estimates. People have characterized the formula as "breaking down." The criticism is invalid because no one has shown that the formula actually "breaks." But since DLOM cannot be greater than 100 percent, the logical solution is to limit Longstaff DLOM results to 100 percent. That approach is taken in this article when applying the described probability-based analysis. This topic is discussed fully in *Empirical Research,* chapter 7.

18 The Pluris and FMV/Stout databases do not provide the standard deviations of the issuers' stock price volatilities.

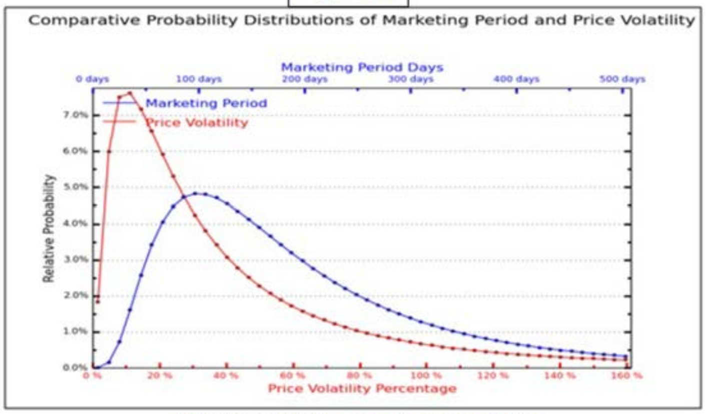
I then reconsidered the 59-transaction population using 250 (or the maximum available) trading days of average price volatility preceding each transaction closing date, and the assumed 90-, 180-, and 360-day marketing periods. Average price volatility was calculated using the VFC DLOM Calculator. The standard deviation of price volatility was ignored (see Table 5).[19]

Table 5 shows that these tests resulted in R-squares of correlation ranging from of 51.57 percent to 59.17 percent. Eight of the regression results had x-y intercepts ranging from positive 0.84 percent to minus 2.8 percent.[20] These results meet two of the parameters for potentially falsifying the Formulas Null Hypothesis. However, only one test successfully falsified the hypothesis with all three parameters: Black-Scholes using 360 days as the marketing period assumption, shown in bold type in Table 5. This test resulted in a coefficient of x of 1.0022, an x-y intercept of –2.54 percent, and an R-square of correlation of 57.21 percent[21] (see Graph 1). It is error to say that the Black-Scholes formula with 0 percent assumed for the risk-free rate and the dividend yield variables, 250 preceding trading days of price volatility, and an assumed 360-day time period, did not reliably predict the set of corresponding restricted stock discounts.



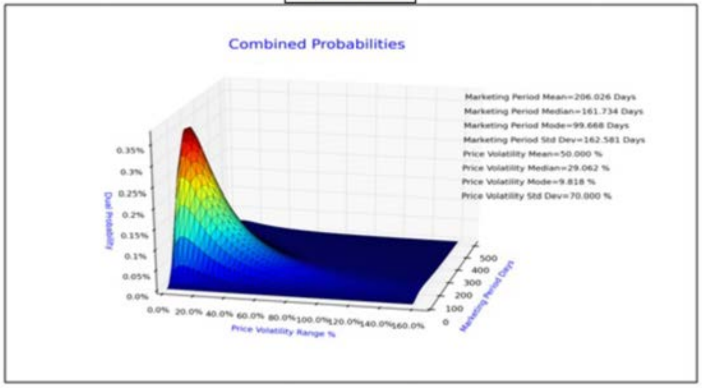
Adding probability to the time and price volatility variables of the Finnerty, Black-Scholes, and Longstaff formulas makes a material difference in their results.[22] The crux of this concept is that we do not live in a static world. Instead, everything is constantly changing, which applies to the price volatility and marketing period variables of the three formulas. It is therefore illogical to use constant values for those variables when trying to calculate the DLOM applicable to an investment that does not have a fixed price or a fixed time period to the liquidity event. As discussed in *Empirical Research*, I surmised that probability could be built into the formulas' DLOM results by

Figure 6.22



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 6.23



Source: VFC DLOM Calculator, www.dlomcalculator.com

determining both the mean and standard deviation of the price volatility and time period variables, spreading the means over the range of outcomes (the VFC DLOM Calculator assumes 50 statistical buckets determined by the standard deviations), calculating a separate DLOM for each combination of price volatility and time (marketing period), applying the probability of occurrence to each datapoint (totaling 100 percent), and summing the resulting DLOMs. Fifty price volatility points times 50 marketing period points is 2,500 datapoints. The distributions of price volatility and time might separately look like *Empirical Research* Figure 6.22. Combined, those distributions look like *Empirical Research* Figure 6.23.

The cone in Figure 6.23 represents the combinations of price volatility and time that are most likely to occur, but any combination shown in the graph has a chance of occurrence. The entire area of the graph represents 100 percent of the potential outcomes. Calculating DLOM using the Figure 6.23 concept results in probability-based calculations using the Finnerty, Black-Scholes, and Longstaff formulas. As the means and standard deviations of the price

19 *See* Exhibit 2 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) for the VFC-calculated price volatility means and standard deviations; *see* Exhibit 3 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) for the calculated DLOMs and regression statistics of these tests. All were statistically significant.

20 The ninth test had an x-y intercept of minus 4.27 percent. I find this too distant from "x-y intercept ~ zero."

21 The eight other tests had coefficients of x at or below 0.7149 or at or above 1.3517. I find these too distant from "coefficient of x ~ y."

22 See *Empirical Research*, chapters 6, 7, and 8.

volatility and time source data change, so too does the distribution of the combined probability.

I also surmised that the expected time to obtain SEC approval for the restricted stock transaction might have affected the observed discounts. Accordingly, in addition to the mean and standard deviation of the issuers' closing stock prices for the 250 trading days preceding the closing date of each stock transaction, the means and standard deviations of the SEC approval periods for the issuers' one- to four-digit SIC codes for up to 10 years preceding the closing dates were calculated using the VFC DLOM Calculator. DLOMs were then calculated for each formula, using (a) only average price volatility and only average SEC processing time; (b) only the average price volatility, but probability-based SEC processing times; and (c) probability-based price volatility, but only average SEC processing time. The tests were run using the four-digit SIC code means and standard deviations as described in Table 6.[23] These tests resulted in R-squares of correlation ranging from 54.66 percent to 62.81 percent. The x-y intercepts of these tests ranged from positive 0.58 percent to negative 3.02 percent.[24]

Table 6: Regression Results for 59 Pluris and FMV/Stout Restricted Stock Transactions Using Combinations of VFC-Calculated Price Volatilities and SEC Processing Times with/without Probabilities

	Formula	R-Square
Finnerty: PV 250 NASDAQ mean STDDEV = 0; SEC processing 4-digit SIC code mean, StdDev = 0	y = 5.1829x - 0.0047	R^2 = 0.608
Black-Scholes: PV 250 NASDAQ mean STDDEV 0; SEC processing 4-digit SIC code mean, StdDev = 0	y = 1.6943x - 0.0131	R^2 = 0.6133
Longstaff: PV 250 NASDAQ mean STDDEV 0; SEC processing 4-digit SIC code mean, StdDev = 0	y = 0.6658x + 0.0058	R^2 = 0.6281
Finnerty: PV 250 NASDAQ mean STDDEV 0; Marketing Period Probability 4-digit SIC code	y = 5.4097x - 0.0068	R^2 = 0.5997
Black-Scholes: PV 250 NASDAQ mean STDDEV 0; Marketing Period Probability 4-digit SIC code	y = 1.7994x - 0.0111	R^2 = 0.6036
Longstaff: PV 250 NASDAQ mean STDDEV 0; Marketing Period Probability 4-digit SIC code	y = 0.7467x + 7E-05	R^2 = 0.6185
Finnerty: Price Volatility Probability; SEC processing 4-digit SIC code mean, StdDev = 0	y = 6.3251x - 0.0168	R^2 = 0.5501
Black-Scholes: Price Volatility Probability; SEC processing 4-digit SIC code mean, StdDev = 0	y = 1.9526x - 0.0231	R^2 = 0.5835
Longstaff: Price Volatility Probability; SEC processing 4-digit SIC code mean, StdDev = 0	**y = 0.9429x - 0.0302**	**R^2 = 0.5466**

The Table 6 R-square and intercept ranges reasonably meet two of the parameters of the Formulas Null Hypothesis. However, only one test reasonably satisfied the coefficient of x parameter: VFC Longstaff using probability-based price volatility and average (i.e., static) SEC processing times for the applicable SIC codes (shown in bold type in Table 6). This test resulted in a coefficient of x of 0.9429, an x-y intercept of 3.02 percent, and an R-square of correlation of 54.66 percent (see Graph 2). This test reasonably falsified the Formulas Null Hypothesis, although I prefer that the x coefficient be within +/– 5 percent of 1.0. With that caveat, it is error to say that the VFC Longstaff method using company-specific price volatility probabilities and industry-specific SEC average processing times did not predict the set of corresponding restricted stock discounts.[25] Because the Longstaff formula was crafted specifically to estimate DLOM, it would also be error to say that this method did not reliably predict DLOM.

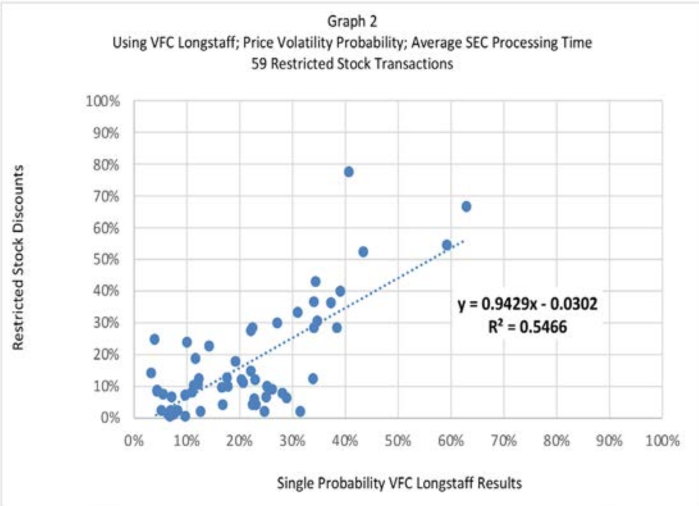
Finally, I considered double-probability calculations of the Finnerty, Black-Scholes, and Longstaff formulas, meaning that the tests were performed using the means and standard deviations of both price volatility and SEC processing time pursuant to the Figure 6.23 concept.[26] Although their R-squares of correlation and x-y intercepts were acceptable, the Finnerty and Black-Scholes formulas failed their tests of the Formulas Null Hypothesis with coefficients of x equal

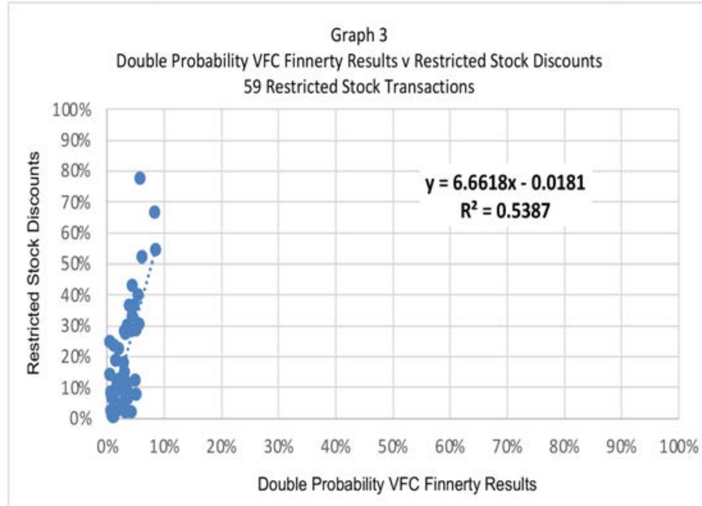
23 *See* Exhibit 2 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) for the details.

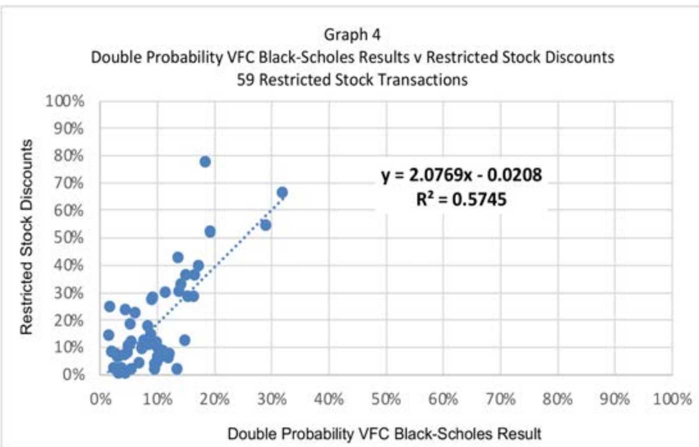
24 *See* Exhibit 4 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) for the calculated DLOMs and regression statistics of these tests. All were statistically significant.

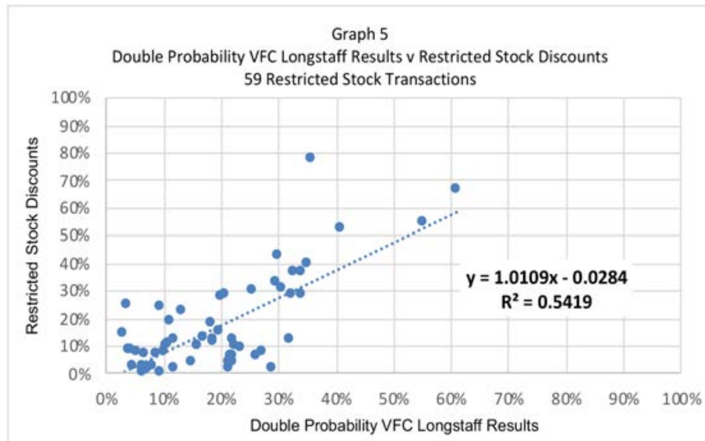
25 The VFC Longstaff method limits Longstaff formula DLOMs to 100 percent. See *Empirical Research*, chapter 7 for more information.

26 *See* Exhibit 5 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) for the calculated DLOMs and regression statistics of these tests. All were statistically significant.









to 6.6618 (Graph 3) and 2.0769 (Graph 4), respectively. These coefficients mean that the Finnerty and Black-Scholes formulas, which were not crafted for DLOM estimation, understated DLOM by about 85 percent and 50 percent, respectively.

Exhibit 5[27] shows that the calculated DLOMs using the Finnerty formula range from 2.80–3.01 percent, and using the Black-Sholes formula they range from 9.11–9.36 percent. In contrast, the average reported discount of the 59-transaction population is 16.85 percent. It is unsubstantiated to say that the Finnerty and Black-Scholes formulas reliably predict DLOM.

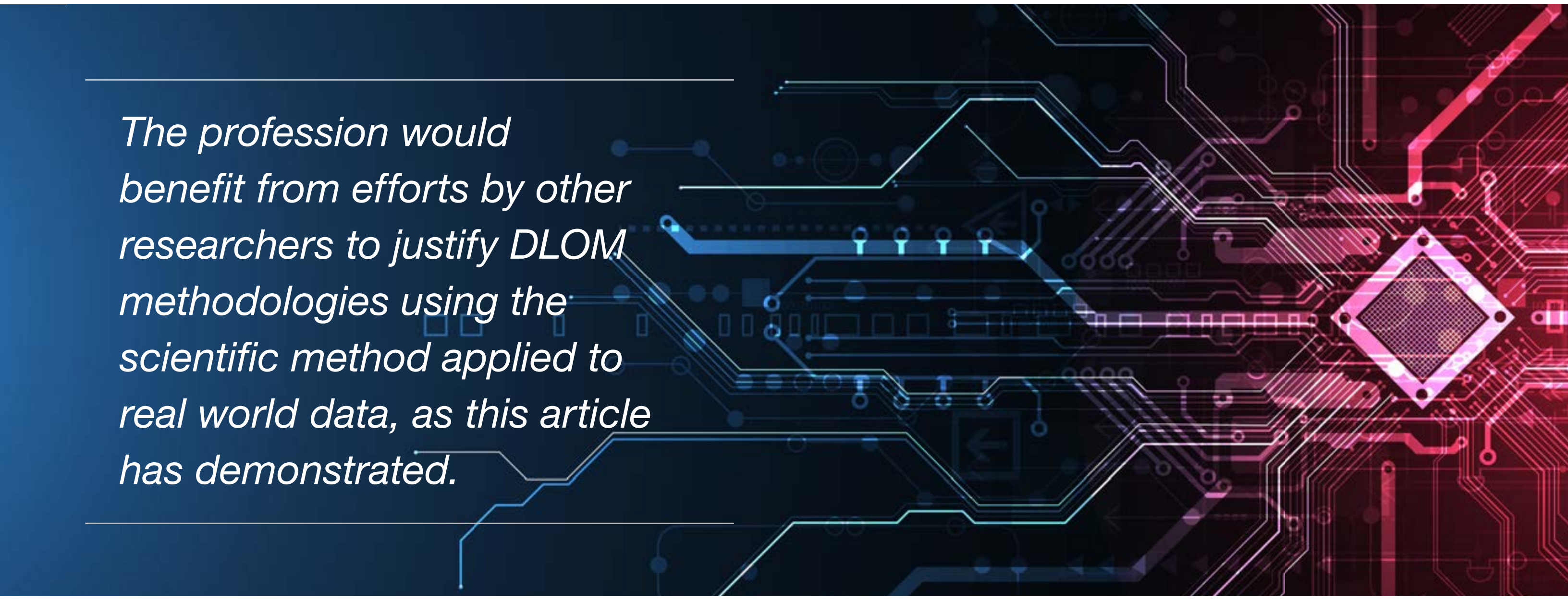
The double-probability VFC Longstaff methodology predicted DLOM consistent with the discounts of the corresponding restricted stock transactions, resulting in an x-y intercept of minus 2.84 percent, a coefficient of x of 1.0109, and an R-square of correlation of 54.19 percent. The test successfully falsified the Formulas Null Hypothesis. The double-probability Finnerty and Black-Scholes tests failed to falsify the hypothesis, which adds credence to the reliability of the VFC Longstaff method for DLOM estimation.[28] It is error to say that the double-probability VFC Longstaff method did not reliably predict the set of corresponding restricted stock discounts and DLOM.

Exhibit 5 shows that the averages of the double probability DLOMs calculated using the VFC Longstaff methodology range from 19.48–19.96 percent. In contrast, the average reported discount of the 59-transaction population is 16.85 percent.[29]

27 https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf.

28 The Formulas Null Hypothesis was falsified by the double-probability VFC Longstaff method regardless of the number of SIC code digits used to estimate the means and standard deviations of the SEC processing times for the 59-transaction population of restricted stocks. The double-probability Finnerty and Black-Scholes formula tests failed to falsify the hypothesis regardless of the number of SIC code digits used to estimate the means and standard deviations of the time variable. Exhibit 2 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) shows the shifts in means and standard deviations of the SEC processing periods as the population of issuers is refined from one to four SIC code digits. Exhibit 5 (https://thevalueexaminer.com/2022/22-SO/22-SO-Vianello-exhibits.pdf) shows the resulting shifts in calculated DLOMs.

29 In *Empirical Research*, the discussion of Table 8.4 explains that further limiting the 59-transaction restricted stock population by removing those with discounts of 5 percent or less, a reduction to 45 transactions, results in an average restricted stock discount of 21.40 percent and an average double-probability VFC Longstaff DLOM of 21.61 percent.



The profession would benefit from efforts by other researchers to justify DLOM methodologies using the scientific method applied to real world data, as this article has demonstrated.

Conclusion and Suggestions for Further Research

One set of assumptions used with the Black-Scholes formula and five sets of assumptions used with the VFC Longstaff method falsified the hypothesis, “The probability-based formula *does not predict* the corresponding observed restricted stock discounts with a linear regression relationship exhibiting these characteristics: (1) x-y intercept ~ zero; (2) coefficient of x ~ y; and (3) R-square > 50 percent.” All other DLOM methodologies discussed in this article failed to falsify the applicable null hypothesis, and should be considered unreliable until scientifically shown otherwise.

It is error to say that the Black-Scholes test depicted by Graph 1 did not reliably predict the corresponding restricted stock discounts. However, because the Black-Scholes formula was not crafted to predict DLOM, no conclusion can be reached about its ability to predict DLOM.

It is error to say that the VFC Longstaff tests depicted by Graph 2 and Graph 5 and shown in Exhibit 5 do not reliably predict the corresponding restricted stock discounts. Additionally, the Longstaff formula was specifically crafted to estimate DLOM. Therefore, falsifying the Formulas Null Hypothesis provides substantiation that the probability-based VFC Longstaff method predicts DLOM.

The single-probability VFC Longstaff test (Graph 2) is somewhat rigid, with DLOM calculations based on a static time period. That may be appropriate in some applications, such as a fixed-term lockup of a security. My preference for the typical fair market valuation of a nonmarketable security is to use the double-probability VFC Longstaff approach (Graph 5 and Exhibit 5) because it is dynamic. It more accurately reflects the world in which investments are made.

As shown above, there are significant issues regarding DLOM measurement and reliable DLOM estimation. The valuation profession would benefit from more extensive access to historical stock prices than were available to me. Access to significantly more price history would allow me to consider a greater population of transactions in the Pluris and FMV/Stout databases. The analysis discussed in this article would also be enhanced if a researcher undertook the task of repricing the Pluris transactions with attached warrants using the Black-Scholes option pricing formula. Finally, the profession would benefit from efforts by other researchers to justify DLOM methodologies using the scientific method applied to real world data, as this article has demonstrated. A simulation-derived analysis of DLOM estimation using the LEAPS methodology is suggested. VE

Marc Vianello, CPA, ABV, CFF, became keenly interested in the topic of discounts for lack of marketability (DLOM) in 2007, which led to the development of the concept of probability-based DLOM. The concept evolved over the years, culminating in the release of his peer-reviewed book, Empirical Research Regarding Discounts for Lack of Marketability. *The research is available as a free e-book from* The VFC DLOM Calculator *website (https://dlomcalculator.com). Mr. Vianello lives and works in Prairie Village, Kansas. Email: vianello@vianello.biz.*