Rebutting Critics of the Longstaff DLOM Methodology

By Marc Vianello, CPA, ABV, CFF

In 1995, when Francis A. Longstaff, Ph.D., presented his idea that the formula for calculating the value of a lookback option with and without a liquidity restriction assumption could be used to estimate the discount for lack of marketability (DLOM) of a financial instrument, he described his approach as quantifying the cost of illiquidity for an investor with otherwise perfect market timing ability. But Dr. Longstaff also recognized that the value of marketability, and, therefore, the cost of illiquidity, is less for investors with less-than-perfect market timing ability. Consequently, Dr. Longstaff described his approach as the "upper bound" of DLOM calculations. Since 1995, criticisms of what is now known as the Longstaff methodology have focused on three perceived defects: 1) no investor has perfect knowledge; 2) a DLOM based on an upper bound is excessive; and 3) the lookback option formula "breaks down" with long marketing periods and high price volatilities. Each of these criticisms is wrong for the reasons described below.

The 'perfect knowledge' criticism. The "perfect knowledge" criticism is based on a defective definition of market timing in a valuation context. The context considered by Dr. Longstaff was one of an investor looking back in time to observe precisely when an investment could have been sold at its maximum value. Dr. Longstaff implicitly assumed that the maximum price could have been reached at any point during the lookback period. But in a valuation context, this reasonable assumption is not appropriate. Instead, the maximum price occurs on the

valuation date and is the marketable value of the valuation subject. Appraisers determine this value in the ordinary course of their work.

Standing on the vantage point of the valuation date and applying a lookback option pricing to calculate DLOM in a business valuation inherently assumes that the maximum price that the investor could have realized is the marketable equivalent price as of that date. The value of the investment beyond the valuation date is necessarily less. This is because the time value of money diminishes the present value of the marketable equivalent price over the course of the marketing period, the foreseeable favorable events affecting the valuation subject have been factored into the analysis, and investors are averse to the risks of price volatility. Thus, if the appraiser properly determined the marketable equivalent price as of the valuation date, then that price is the "maximum value" postulated by Dr. Longstaff.

The 'upper bound' criticism. Dr. Longstaff described the framework in which an upper bound on the value of marketability is derived as one lacking the assumptions about informational asymmetries, investor preferences, and other variables that would be required for a general equilibrium model. Dr. Longstaff recognized that the cost of illiquidity is less for an investor with imperfect market timing than it is for one possessing perfect market timing. These considerations are the basis of the upper bound limitation of the Longstaff methodology.

It is irrefutable that the cost of illiquidity must be less for the average investor with imperfect market timing than it is for an investor possessing perfect market timing. But the upper bound criticism resulting from this situation is nonetheless defective in the valuation context because it is easily circumvented by using volatility estimates that represent average, not peak, volatility expectations. For example, the appraiser's volatility estimate may be based on some average or regression of historical price volatility derived from an index or from one or more publicly traded guideline companies. Using average volatility estimates in the lookback option formula necessarily results in a value that is less than the upper bound value. Indeed, a value calculated using average expected volatility necessarily suggests a result that is achievable by the average imperfect investor. The resulting value determined in this manner appropriately falls short of a value based on perfect market timing while providing an important informational asymmetry lacking in Dr. Longstaff's more simplified framework.

Enhanced estimates of DLOMs applicable to average investors can also be crafted by determining the average marketing period required to sell privately held businesses and the standard deviation of distribution around the mean.[1] Using probability weighted marketing periods, therefore, provides a second important informational asymmetry lacking in Dr. Longstaff's framework.

Additional framework enhancements include determining the rate of incline or decline in future volatility and weighting future volatility estimates according to the probability of sale associated with the period in which the estimates are expected to occur. Accordingly, the upper bound criticism has no significance in a proper application of the Longstaff methodology.

---

1 E.g., Vianello, "The Marketing Period of Private Sale Transactions: Updated for Sales Through 2010," *Business Valuation Update*, November 2011.

Reprinted with permissions from Business Valuation Resources, LLC

The 'formula breaks down' criticism. The formula presented in Dr. Longstaff's 1995 article is:

$$Upper\ Bound = V\left(2 + \frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + V\sqrt{\frac{\sigma^2 T}{2\pi}}\ \exp\left(-\frac{\sigma^2 T}{8}\right) - V$$

where:

V = current value of the investment

σ = volatility

T = marketability restriction period

N = standard normal cumulative distribution function

The IRS publication *Discount for Lack of Marketability—Job Aid for IRS Valuation Professionals* makes the statement that volatilities in excess of 30% are not "realistic" for estimating DLOM using lookback option pricing models. In support of this contention, the publication provides a table reporting marketability discounts in excess of 100% resulting from using combinations of variables of at least 50% volatility with a five-year marketing period and 70% volatility with a two-year marketing period. When that occurs, the Longstaff DLOM should simply be capped at 100%. After all, the criticism is not that the formula incorrectly calculates DLOMs below the 100% limit—merely that DLOM cannot exceed 100%.

Exhibit 1 shows the Longstaff DLOMs, capped at 100%, that result from a 20% price volatility assumption and a broad range of marketing periods. The 20% price volatility assumption approximates the historical mean of the VIX from Jan. 2, 1990, to June 30, 2011. Note that it takes about 6,970 days—over 19 years—for the discount to reach 100% with a 20% price volatility assumption. Considering that the typical business sells in about 200 days, a criticism based on a 19-year marketing period is clearly unreasonable.[2]
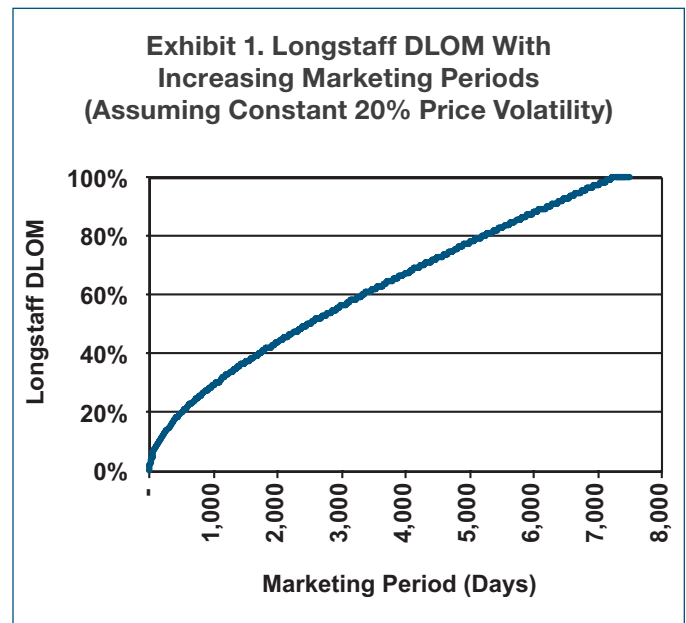
Of course, as the expected price volatility increases, a shorter time is required to reach 100%. Conversely, as the expected price volatility decreases, a longer time is required to reach 100%. Exhibit 2 shows the line demarking varying combinations of price volatility and marketing periods above which Longstaff DLOMs exceed 100%. Considering that the peak volatility of the VIX was about 80% (occurring on Nov. 20, 2008) and that the average period in which a private business sells is about 200 days, it is unlikely that typical appraisers will define lookback option variables that result in Longstaff DLOMs that exceed 100%.

Some appraisers may nonetheless struggle with the idea of using a formula to calculate DLOM that "breaks down" under certain assumptions. The dilemma is avoided by applying the formula adjusted DLOM = average DLOM/(1 + average DLOM) that I suggested in my 2009 article, "Calculating DLOM Using the VFC Longstaff Methodology."[3] This adjustment assures that even with the highest volatilities and longest marketing periods, DLOM never exceeds 100%.

For example, the IRS publication reports a discount percentage of 106.7% based on an

---

3   *BVR's Guide to Discounts for Lack of Marketability, 2009.*

Exhibit 1. Longstaff DLOM With Increasing Marketing Periods (Assuming Constant 20% Price Volatility)

---

2   The VIX peaked at 80.86% on Nov. 20, 2008. With that assumption, the Longstaff formula requires a 450-day lockup period to reach 100% DLOM.
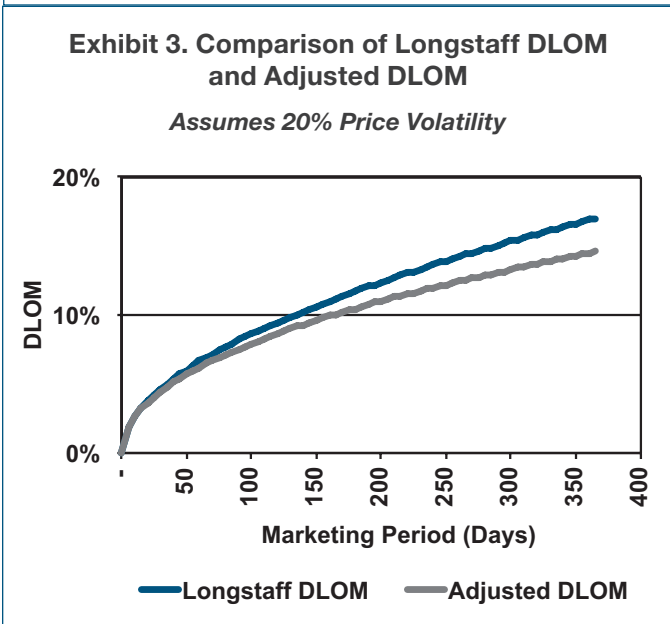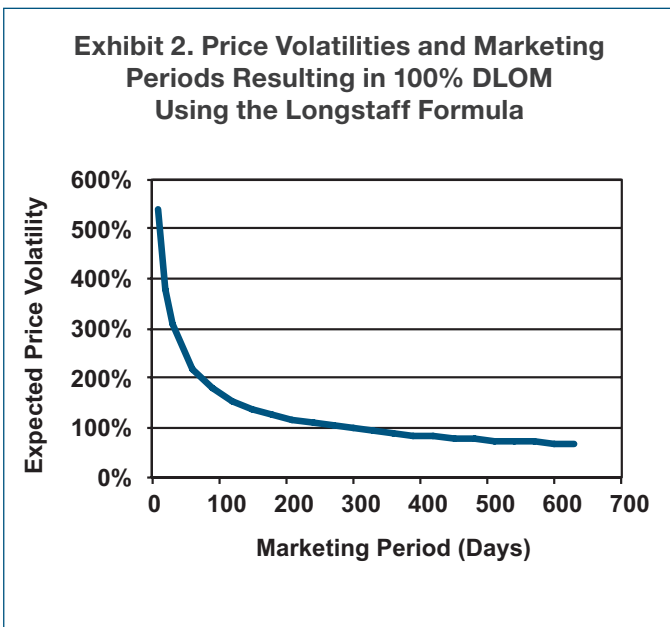
Reprinted with permissions from Business Valuation Resources, LLC

estimated 70% price volatility over an estimated two-year post-valuation date marketing period. The DLOM percentage resulting from the same parameters and using the above technique is 51.6%. This modification of the Longstaff method makes it mathematically impossible for the resulting percentage to equal or exceed 100% of the marketable value of the valuation subject. But *adjusted* DLOM increasingly understates Longstaff DLOM as the marketing period assumption lengthens and the price volatility assumption elevates.

Exhibit 2. Price Volatilities and Marketing Periods Resulting in 100% DLOM Using the Longstaff Formula

Exhibit 3. Comparison of Longstaff DLOM and Adjusted DLOM

Assumes 20% Price Volatility

Conclusion. As discussed in the first two sections of this article, the "perfect knowledge" and "upper bound" criticisms of the Longstaff methodology are meritless. Additionally, it has not been shown that the accepted formula for pricing lookback options is somehow inaccurate. Therefore, if appraisers base their analyses on reasonable estimates of the average expected price volatility and the average expected marketing period for the valuation subject, the appropriate conclusion is that Longstaff DLOMs below 100% are reasonable estimates of the cost of illiquidity to the average hypothetical investor. The resulting DLOMs under such conditions are below the upper bound postulated by Dr. Longstaff. Marketing period and price volatility estimates that result in Longstaff DLOMs greater than 100% are most likely unrealistically high. Regardless, the resulting DLOMs should be capped at 100%. *Adjusted* DLOMs calculated with the technique described in the third section of this article may be useful as a "lower bound" estimate of the cost of illiquidity and for those appraisers who remain concerned about the "formula breaks down" criticism (see Exhibit 3).

Because the variables entering into the generally accepted lookback option formula can be objectively determined and verified, the formula can be tailored to specific businesses at specific points in time. Thus, carefully crafted applications of the Longstaff methodology provide appraisers with a powerful tool for estimating (or challenging) discounts for lack of marketability.

Marc Vianello, CPA, ABV, CFF, is the managing member of Vianello Forensic Consulting, LLC in Overland Park, Kan., and may be reached at vianello@vianello.biz.